FOR INCOMPRESSIBLE FLOWS

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Proefproject Massaal Parallel Rekenen

Objective

- To develop solvers for a sparse linear system of equations.
- Properties: iterative, parallel, scalable, and fast.

Motivation

- Computational Fluid Dynamics leads to very large systems of equations.
  For medium sized 3 dimensional problems the time to compute a solution on a fast workstation can be one week or more.
  This motivates the search for scalable parallel algorithms.

Approach

Method

Fastest sequential method is used: preconditioned GMRES where the preconditioners are based on Incomplete LU factorizations.

Decomposition

The rectangular computational domain is divided into strips parallel to the x_2-axis. Every strip is computed on a different processor.

Parallelization

The method consists of the following parts (modified Gram-Schmidt is used):

vector update:
Every processor computes its own part. No communication.

inner product:
After computing the local inner products global communication is necessary (a Cray T3D shared memory array is used).

matrix vector product:
Only nearest neighbour communication is necessary.

preconditioner times vector:
This is the most difficult part to parallelize.

Parallel preconditioning

Preconditioning consists of:

- LU factorization
- $Ly = b$ lower triangular
- $Ux = y$ upper triangular

These parts can be parallelized in the same way.

Consider $Ly = b$. First the elements on the first line in the first strip (processor) are computed. Thereafter communication takes place and the elements on the second line on processor 1 and those on the first line on processor 2 are calculated in parallel, etc. This idea is visualized in Figures 1, 2 and 3.

∗ to be calculated
∗ are calculated
∗ already calculated

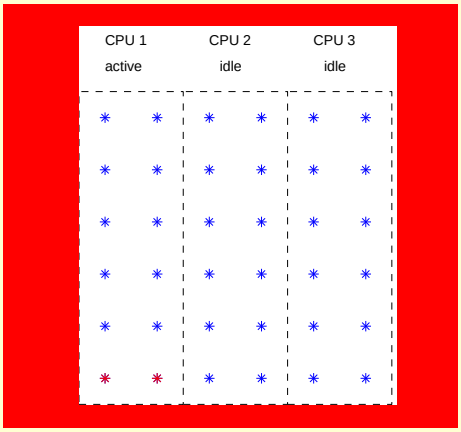

Figure 1 First step

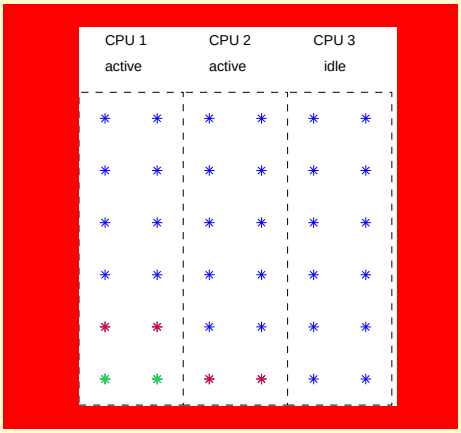

Figure 2 Second step

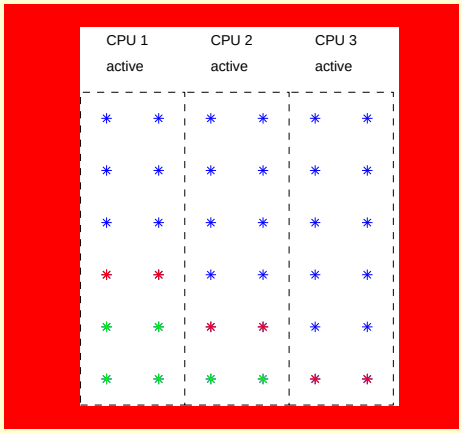

Figure 3 Third step

Results

Used machine: Cray T3D at the Edinburgh Parallel Computing Center.
Results for the momentum equations are given in Figure 4.

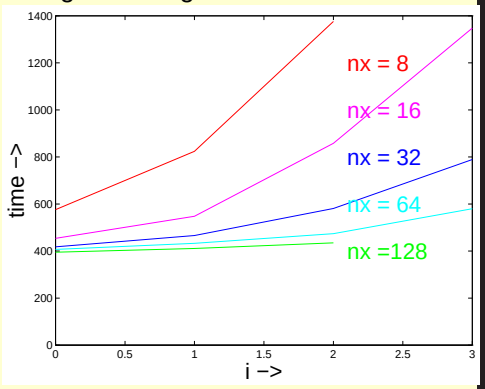

Figure 4 The total CPU time per variable in μ seconds is measured for a fixed number of iterations.

The grid size is: $nx \cdot 2^{i+2} \times nx \cdot 2^i$ and $2^{(2i+1)}$ processors are used. Note that the solver is scalable for nx large enough.

Publications

R.P.P. van Nooyen, C. Vuik and P. Wesseling, Parallellism in GMRES applied to the computation of incompressible flows, TWI Report 96-109, TU Delft, 1996.