

Bi-CG

x_0 is given; $r_0 = b - Ax_0$;

\hat{r}_0 is an arbitrary vector $(\hat{r}_0, r_0) \neq 0$

possible choice $\hat{r}_0 = r_0$;

$$\rho_0 = 1$$

$$\hat{p}_0 = p_0 = 0$$

for $i = 1, 2, \dots$

$$\rho_i = (\hat{r}_{i-1}, r_{i-1}) ; \beta_i = (\rho_i / \rho_{i-1}) ;$$

$$p_i = r_{i-1} + \beta_i p_{i-1} ;$$

$$\hat{p}_i = \hat{r}_{i-1} + \beta_i \hat{p}_{i-1} ;$$

$$v_i = Ap_i$$

$$\alpha_i = \rho_i / (\hat{p}_i, v_i) ;$$

$$x_i = x_{i-1} + \alpha_i p_i$$

$$r_i = r_{i-1} - \alpha_i v_i$$

$$\hat{r}_i = \hat{r}_{i-1} - \alpha_i A^T \hat{p}_i$$

end for

Bi-CGSTAB method

x_0 is an initial guess; $r_0 = b - Ax_0$;

\bar{r}_0 is an arbitrary vector, such that $(\bar{r}_0, r_0) \neq 0$,

e.g., $\bar{r}_0 = r_0$;

$\rho_{-1} = \alpha_{-1} = \omega_{-1} = 1$;

$v_{-1} = p_{-1} = 0$;

for $i = 0, 1, 2, \dots$ do

$\rho_i = (\bar{r}_0, r_i)$; $\beta_{i-1} = (\rho_i/\rho_{i-1})(\alpha_{i-1}/\omega_{i-1})$;

$p_i = r_i + \beta_{i-1}(p_{i-1} - \omega_{i-1}v_{i-1})$;

$\hat{p} = K^{-1}p_i$;

$v_i = A\hat{p}$;

$\alpha_i = \rho_i/(\bar{r}_0, v_i)$;

$s = r_i - \alpha_i v_i$;

if $\|s\|$ small enough then

$x_{i+1} = x_i + \alpha_i \hat{p}$; quit;

$z = K^{-1}s$;

$t = Az$;

$\omega_i = (t, s)/(t, t)$;

$x_{i+1} = x_i + \alpha_i \hat{p} + \omega_i z$;

if x_{i+1} is accurate enough then quit;

$r_{i+1} = s - \omega_i t$;

end for

GCR algorithm

choose x_0 , compute $r_0 = b - Ax_0$

for $i = 1, 2, \dots$ do

$$s_i = r_{i-1} ,$$

$$v_i = As_i ,$$

for $j = 1, \dots, i - 1$ do

$$\alpha = (v_i, v_j) ,$$

$$s_i := s_i - \alpha s_j , \quad v_i := v_i - \alpha v_j ,$$

end for

$$s_i := s_i / \|v_i\|_2 , \quad v_i := v_i / \|v_i\|_2$$

$$x_i := x_{i-1} + (r_{i-1}, v_i) s_i ;$$

$$r_i := r_{i-1} - (r_{i-1}, v_i) v_i ;$$

end for

GMRESR algorithm

choose x_0 and m , compute $r_0 = b - Ax_0$

for $i = 1, 2, \dots$ do

$$s_i = P_{m,i-1}(A)r_{i-1} ,$$

$$v_i = As_i ,$$

for $j = 1, \dots, i - 1$ do

$$\alpha = (v_i, v_j) ,$$

$$s_i := s_i - \alpha s_j , \quad v_i := v_i - \alpha v_j ,$$

end for

$$s_i := s_i / \|v_i\|_2 , \quad v_i := v_i / \|v_i\|_2$$

$$x_i := x_{i-1} + (r_{i-1}, v_i) s_i ;$$

$$r_i := r_{i-1} - (r_{i-1}, v_i) v_i ;$$

end for

