Oil and Gas Production Forecasting with Semi-Analytical Reservoir Simulation

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‘Reservoir simulation’ tools

- Inflow Performance Relation
- Material balance
- Analytical model
- Numerical model

Diffusion equation

$$\frac{\partial p(\mathbf{x},t)}{\partial t} = \eta \nabla^2 p(\mathbf{x},t) \qquad \eta = \frac{k}{\phi \mu c_t}$$

Numerical solution

$$\frac{p^{n+1} - p^{n}}{\Delta t} = \eta \frac{p_{i+1}^{n+1} - 2 p_{i}^{n+1} + p_{i-1}^{n+1}}{\Delta x^2}$$

Analytical reservoir simulation

Diffusion equation

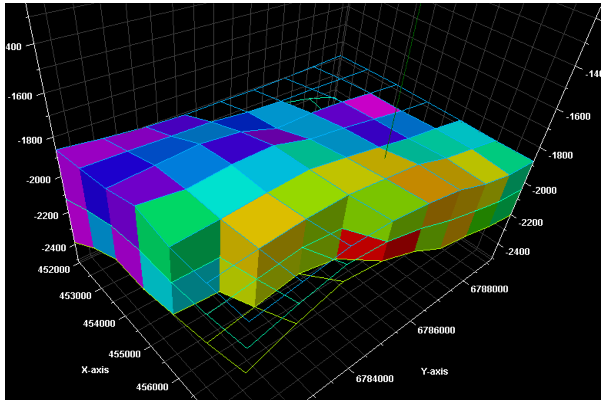
$$\frac{\partial p(\mathbf{x},t)}{\partial t} = \eta \nabla^2 p(\mathbf{x},t) \qquad \eta = \frac{k}{\phi \mu c_t}$$

Analytical solution

$$\Delta p(M, M', t) = \frac{1}{\Phi c} \int_0^t \tilde{q}(\tau) S(M, M', t-\tau)\, \mathrm{d}\tau,$$

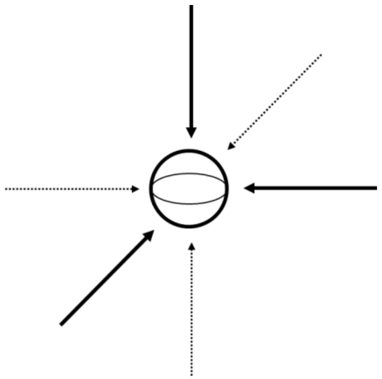
$$S(M, M', t-\tau) = \frac{1}{8\sqrt{\eta_x \eta_y \eta_z}\,[\pi(t-\tau)]^{3/2}} \exp\left[-\frac{(M-M')^2/\tilde{\eta}}{4(t-\tau)}\right]$$

Extension to different types of <u>sources</u>, <u>boundaries</u> and <u>fluid system</u>, <u>multi-well</u>, <u>multi-layer</u>, etc.



- Same PDE
- Analytical solution on upscaled reservoir model
- Fast speed for on-time decision making

Basic solutions



- 3D point source solution

$$p(x,y,z,t) = \frac{U(t-t_0)}{8\pi^{3/2}\phi c_t\sqrt{\eta_x\eta_y\eta_z}}\int_0^{t-t_0}\frac{q(t-t_0-\tau)}{\tau^{3/2}}e^{-\left\{\frac{(x-x_0)^2}{4\eta_x\tau}+\frac{(y-y_0)^2}{4\eta_y\tau}+\frac{(z-z_0)^2}{4\eta_z\tau}\right\}}d\tau$$

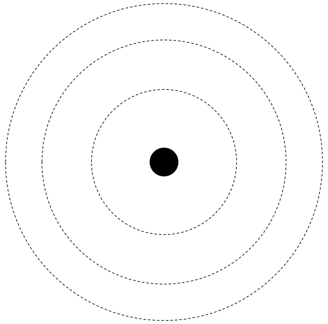
- 2D solution (line source)

$$P_D = \frac{1}{2}\int_0^{t_D}\frac{1}{t'}\exp\left(-\frac{r_D^2}{4t'}\right)dt' = -\frac{1}{2}E_i\left(-\frac{r_D^2}{4t_D}\right)$$

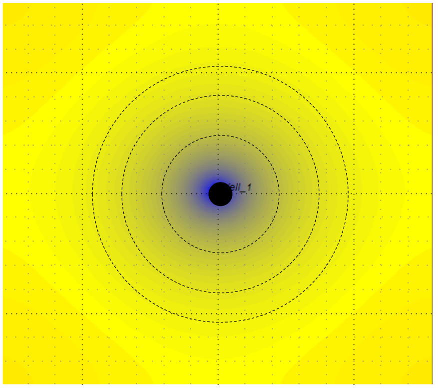
$$r_D = \frac{r}{r_w}$$

$$t_D = \frac{kt}{\phi\mu c_t r_w^2}$$

$$P_D = \frac{2\pi kh\Delta P}{q\mu B}$$



IARF pressure solution



- Infinite Acting Radial Flow

$$p_i - p_{wf}(t, r_w) = \frac{qB\mu}{4\pi kh}\left[\ln t + \ln\frac{k}{\phi\mu c_t r_w^2} + 0.80907\right]$$

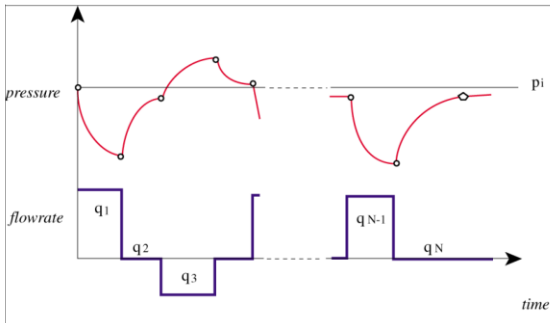
Pressure difference $\Delta p = m \ln t + b$

Pressure derivative $t\frac{\mathrm{d}\Delta p}{dt} = \frac{\mathrm{d}\Delta p}{d\ln t} = m$ *Bourdet derivative*

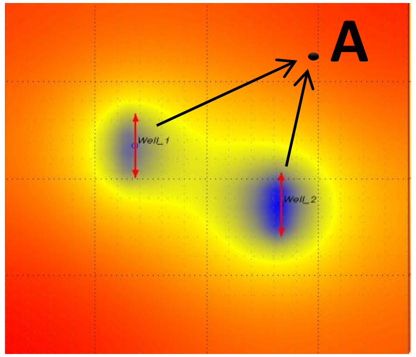
Superposition concept

- **System response to a number of perturbations = sum of responses to each of the perturbations**

- Temporal
  - Multi-rate



- Spatial
  - Multi-well
  - Boundary conditions



R.N. Horne, Modern Well Test Analysis

Pressure equation for multi-phase systems

$$\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = \nabla\cdot\left(\frac{\lambda_o}{B_o}k\nabla p\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) = \nabla\cdot\left(\frac{\lambda_w}{B_w}k\nabla p\right)$$

$$\frac{\partial}{\partial t}\left[\phi\left(R_s\frac{S_o}{B_o}+\frac{S_g}{B_g}\right)\right] = \nabla\cdot\left[R_s\frac{\lambda_o}{B_o}k\nabla p+\frac{\lambda_g}{B_g}k\nabla p\right]$$

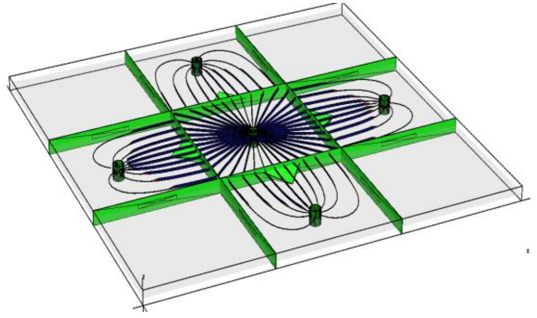
$$\lambda_t\nabla^2 p + k\left(\frac{1}{\mu_o}\frac{dk_{ro}}{dS_o}+\frac{1}{\mu_g}\frac{dk_{rg}}{dS_o}\right)\nabla p\nabla S_o - k\left(\frac{k_{ro}}{\mu_o^2}\frac{d\mu_o}{dp}+\frac{k_{rg}}{\mu_g^2}\frac{d\mu_g}{dp}\right)(\nabla p)^2 = c_t\phi\frac{\partial p}{\partial t}$$

Without the nonlinear terms $\lambda_t = k\left(\frac{k_{ro}}{\mu_o}+\frac{k_{rw}}{\mu_w}+\frac{k_{rg}}{\mu_g}\right)$

$$\lambda_t\nabla^2 p = c_t\phi\frac{\partial p}{\partial t}$$

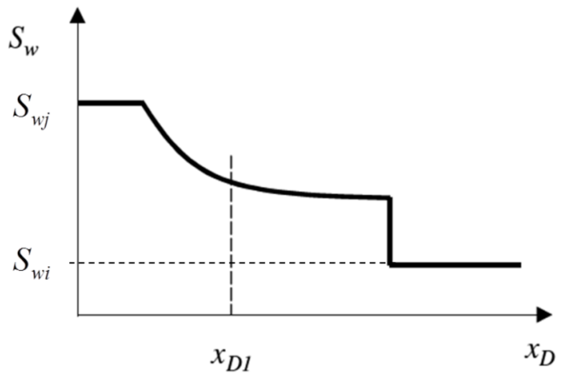
$\lambda_t = \lambda_t(S_o(x), S_w(x), p)$ $c_t = c_t(S_o(x), S_w(x), p)$

Saturation equation

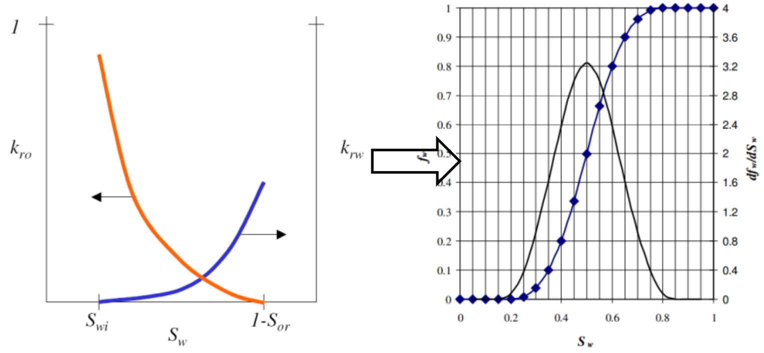


$$\frac{\partial S_w}{\partial t} + v\frac{df_w}{dS_w}\frac{\partial S_w}{\partial x} = 0$$

$$f_w = \frac{\dfrac{k_{rw}(S_w)}{\mu_w}}{\dfrac{k_{rw}(S_w)}{\mu_w} + \dfrac{k_{ro}(S_w)}{\mu_o}}$$

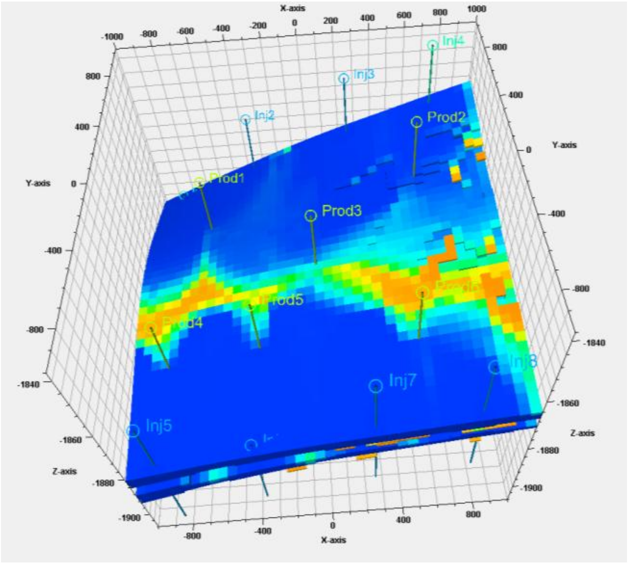


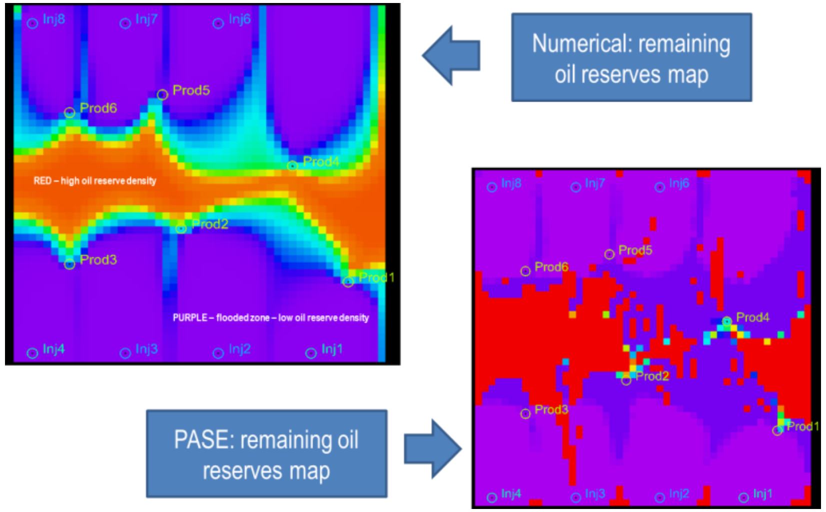
- Streamline constructed from pressure field
- Hyperbolic saturation equation solved along 1D streamlines (analytically or numerically)
- Limited mainly to waterflooding problem

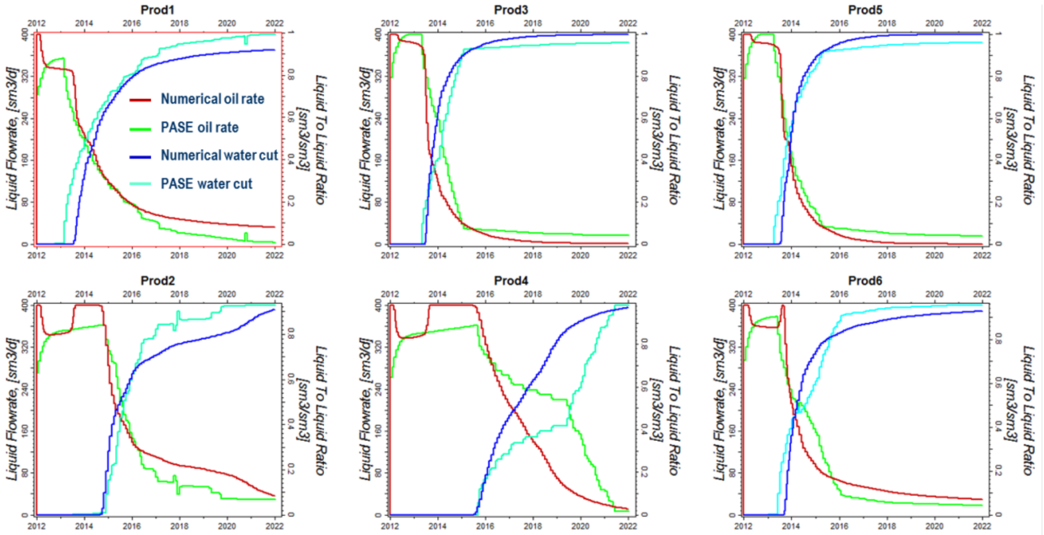


H. Tchelepi, Course note on multi-phase flow in porous media, Stanford University

Waterflooding example

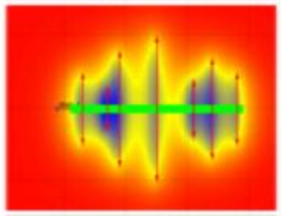
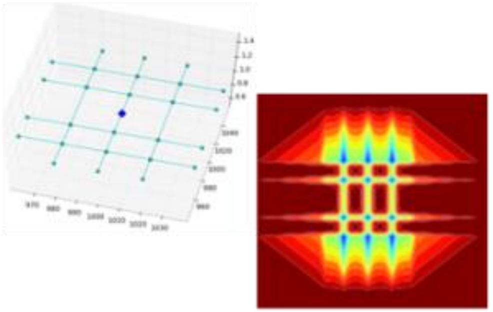
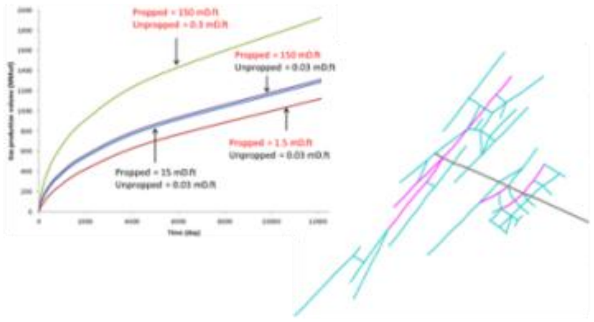






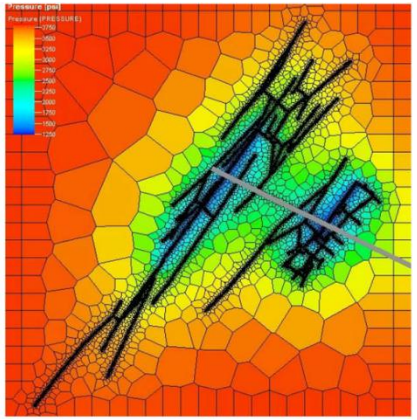
Analytical reservoir simulation for hydraulic fractures

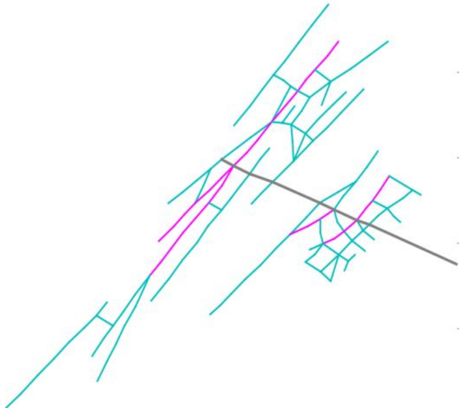
Semi-analytical solution with finite-conductivity fractures
- Analytical reservoir solution: flow from reservoir to fractures
- Numerical fracture solution: flow inside fractures

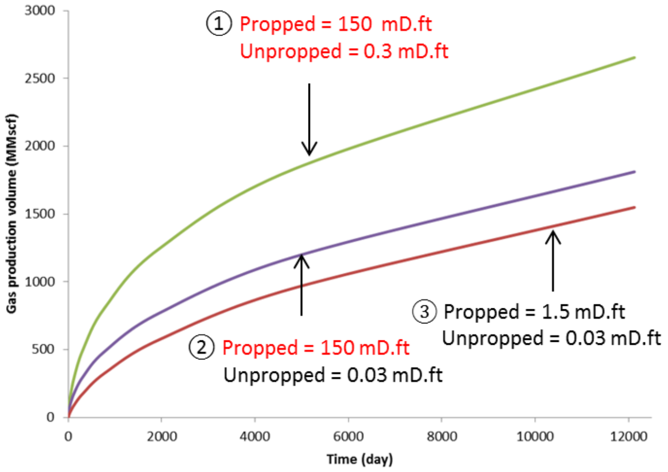
	Planar	Orthogonal FN (Wiremesh)	Complex FN (UFM)
Analytical	Fast simulation, easy-to-use, minimal expertise requirement, accessible		
	 Cinco-Ley, 1981		

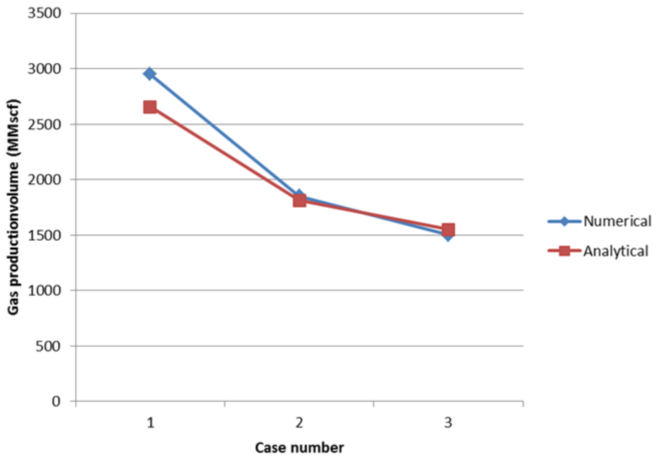
Production simulation of fracture network

- Rigorous HC production simulation from fracture network
- Easy model setup
- Fast simulation
- Easy to use
- Treatment optimization



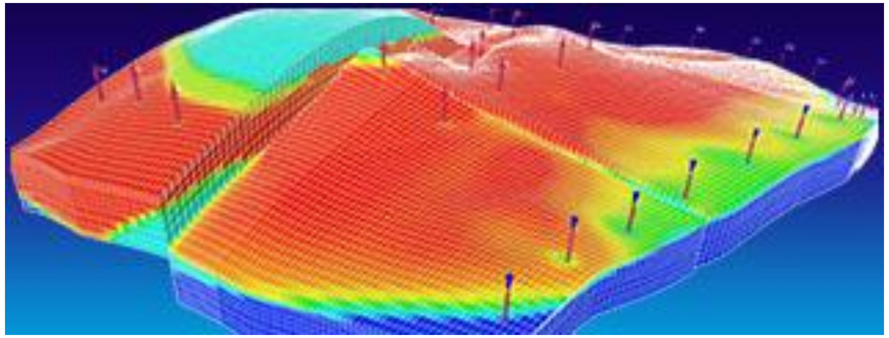


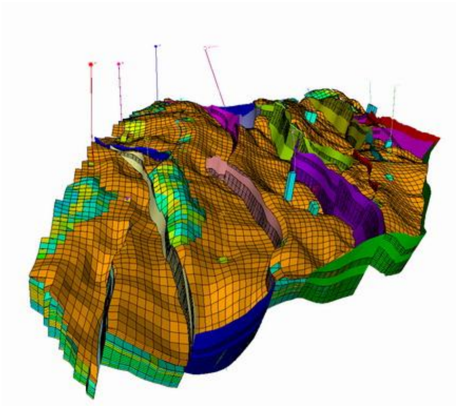


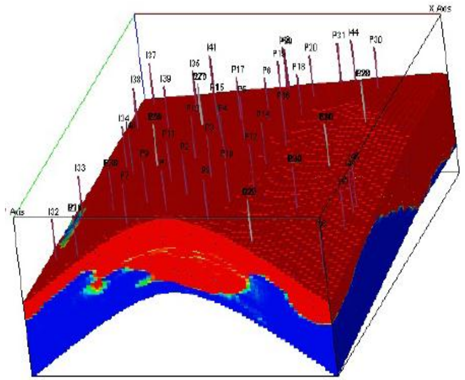


Numerical reservoir simulation

- Multi-phase diffusion equation discretized and solved with finite difference numerical scheme
- Applicable for complex nonlinearities
- Big system (up to million, billion cells)
- Standard tool in the oil & gas industry







Speed of analytical method