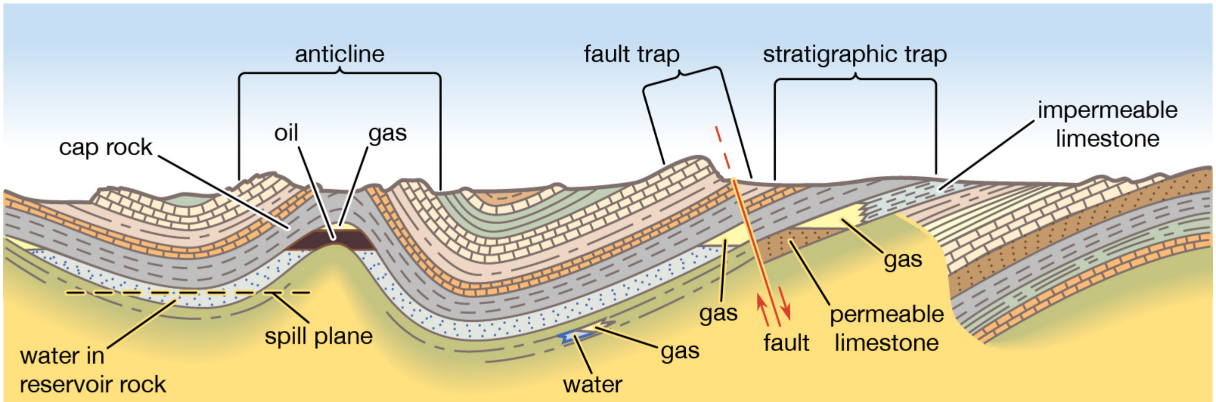
Master Thesis Project proposal: Multi-patch discontinuous Galerkin isogeometric analysis for porous media flow

Introduction

Fluid flows in porous media play an important role in many practical applications such as groundwater flow, subsurface flows, and oil and gas recovery. The overall challenge is to model the interaction of different phases (liquid, gaseous, solid) with the environment that is typically assumed to be inhomogeneous with large variations in porosity, permeability, etc.. Moreover, a typical characteristic of porous media is the presence of layers as illustrated below with large jumps in porosity and permeability between the layers. Largely driven by the interest of the oil and gas industry, sophisticated partial differential equation models have been developed ranging from simple single-phase models to more complex multi-phase models for, e.g., gas, water and oil together with established reservoir simulators mainly based on finite volume method. One of the main difficulties with this (adaptive) unstructured meshing approach is to represent features such as layers and faults by the computational mesh by aligning edges and faces to them.



The basic law governing the flow of fluids through porous media is Darcy's law

Problem description

This project deals with the development of a conceptually new numerical method for porous media flow, and hence, the simplest single-phase model shall be adopted

$$\alpha \frac{\partial(\phi\rho)}{\partial t} = \nabla \cdot \left[\frac{\alpha\rho}{\mu} K(\nabla p + \rho g \nabla D)\right] + \alpha q$$

or its simplified incompressible counterparts assuming that the porosity ϕ is constant

$$0 = \nabla \cdot \left[\frac{\alpha}{\mu} K(\nabla p + g \nabla D)\right] + \frac{\alpha q}{\rho}$$

The aim is to discretize the governing equations using the isogeometric analysis (IgA) approach that was introduced by Hughes et al. [Hu05] mainly for solid mechanical engineering applications. IgA was originally designed with the aim to represent CAD geometries exactly by using the same Spline/NURBS basis functions for modelling geometry and representing the solution. In this project, this ability of IgA shall be exploited by representing the different layers in the porous medium by grid lines. Another strength of IgA is its ability to easily generate high-order approximations of arbitrary order p and to vary the inter-element continuity locally (along grid lines) between C^{p-1} and C^{-1}, which shall be exploited to represent the discontinuities between different layers. The discontinuities caused by faults shall be represented with the aid of the discontinuous Galerkin (dG) method, which will serve as outer coupling for multi-patch IgA [La14].

Challenges

The main challenges of this project are twofold: Firstly, the complex layered structure makes it difficult to generate accurate multi-patch representations of the domain with patch-boundaries and also internal grid lines aligned with the discontinuities. Secondly, the large inhomogeneity present in the permeability tensor leads to stability problems of the discretization, which might require special stabilization techniques, and to severe difficulties in solving the discretized system. This project will focus on the first aspect.

Time schedule

The following tasks are foreseen:

- Literature study on isogeometric analysis, discontinuous Galerkin methods and a brief introduction into single-phase porous media flow.
- Development and implementation of a single-patch 2D-IgA-solver for single-phase porous media flow with the ability to align grid lines with layers.
- Development and implementation of a two-patch 2D-dG-IgA-solver for single-phase porous media flow as a proof-of-concept.
- Extension to multi-patch 2D-dG-IgA-solver and validation for benchmarks.
- Thesis writing.

Contact

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Literature

[Hu05] T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs: Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. Comput. Methods Appl. Mech. Engrg. 194 (2005) 4135-4195.

[La14] U. Langer, A. Mantzaflaris, S.E. Moore, and I. Toulopoulos: Multipatch discontinuous Galerkin isogeometric analysis. arXiv:1411.2478