

NUMERICAL LINEAR ALGEBRA  
ACADEMIC YEAR 2007-2008

**Theoretical assignment day 1**

1. Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}.$$

- What is the Range of  $A$ ?
  - What is the Null space of  $A$ ?
  - What is the rank of  $A$ ?
  - What are the eigenvalues of  $A$ ?
2. Proof that the condition number of an orthogonal matrix is equal to one. Also proof that orthogonal transformations preserve the 2-norm of a matrix.
3. Show that for  $p$ -norms

$$\|AB\|_p \leq \|A\|_p \|B\|_p.$$

4. Proof that the Frobenius norm a matrix norm is.
5. Proof that the induced norms are matrix norms.
6. Show that  $fl(AB) = AB + E$  with  $|E| \leq nu|A||B| + O(u^2)$