NUMERICAL LINEAR ALGEBRA
ACADEMIC YEAR 2009-2010

Theoretical assignments day 8

1. Derive CGLS from CG.

2. Gauss-Seidel for the normal equations (de la Garza's method) defines updates $x_{new} = x_{old} + \delta e_i$, where e_i is the i-th basisvector. δ is selected to minimise the residual norm of x_{new}.
   Write down the resulting algorithm.

3. Overdetermined systems can also be solved by normal equations of the form

$$AA^H y = b \quad x = A^H y$$

   - Show that this system only has a solution if $b \in \mathcal{R}(\mathcal{A})$.
   - Show that x is the minimum norm solution if a solution to this system exists.
   - Derive a CG variant for these normal equations.

4. Consider the damped least squares problem

   - How are the singular values of the damped matrix related with the original matrix?
   - Explain why the damped least squares problem is less sensitive to noise.

5. Prove that CG applied to the Normal Equations leads to x_i for which $\|b - Ax_i\|_2$ is minimised over the (shifted) Krylov subspace $x_0 + \mathcal{K}^k(A^H A; A^H r_0)$.