

PhD-course on Iterative Methods for Linear Systems of Equations

Theoretical assignments day 1

1. Show that the inner product

$$(x, y)_A = x^T A y$$

is a proper inner product if A is Symmetric and Positive Definite.

2. Show that for A symmetric and positive definite the optimal parameter α for Richardson's method is given by $\frac{2}{\lambda_{min} + \lambda_{max}}$.
3. Show how you can estimate the spectral radius of $G = M^{-1}R$ from the iteration process. Show that if $\rho(M^{-1}R)$ is known, an estimate for the error is given by $\|x - x_k\|_2 \leq \frac{\rho(G)}{1 - \rho(G)} \|x_k - x_{k-1}\|_2$.
4. The "least-squares" Gauss-Seidel defines updates $x_{new} = x_{old} + \delta e_i$, where e_i is the i -th basisvector. δ is selected to minimise the residual norm of x_{new} .
Write down the resulting algorithm.