PhD-Course on Iterative Methods for Linear Systems of Equations

Theoretical assignments day 5

1. Show that the eigenvalues of the preconditioned matrix $M^{-1}A$ are solutions of the generalised eigenvalue problem $Ax = \lambda Mx$.

2. Show that the preconditioned matrices $M^{-1}A$, $L^{-1}AU^{-1}$ with $M = LU$, and AM^{-1} have the same spectrum.

3. On parallel computers we want to split the computations into large independent portions of computations. Why is this requirement difficult to combine with a good preconditioner?

4. On day 1 we saw that the inverse of a matrix can be approximated with the power series $\sum_{i=0}^{p}(I-A)^i$. We can use this series as a (polynomial) preconditioner. Is this a good idea in combination with CG? And in combination with GMRES?