Chapter 14
Ten Open Problems in Rendezvous Search

Steve Alpern

Abstract The rendezvous search problem asks how two (or more) agents who are lost in a common region can optimize the process by which they meet. Usually they have restricted speed (unit speed in the continuous time context; moves allowed to adjacent nodes in discrete time). In all cases the agents are not aware of each other's location. This chapter is concerned with the 'operations research' version of the problem – where optimization of the search process is interpreted as minimizing the expected time to meet, or possibly maximizing the probability of meeting within a given time. The deterministic approaches taken by the theoretical computer science community will not be considered here.

14.1 Introduction

A precursor of the rendezvous problem is the work of Schelling [32] on the coordination of meeting places (focal points). However these one-shot games lack the main ingredient of search, namely that the process continues after each failure to meet until (hopefully) the rendezvous is achieved. The rendezvous search problem was first proposed by the author at the end of a talk on search games given in Vienna in 1976 [1].

We shall be mainly concerned with the so called *symmetric*, or *player–symmetric* form of the problem, where both players (agents) must adopt the same rendezvous strategy, though when using mixed strategies they must randomize independently. This version is said to have *indistinguishable agents*. For example if two players, Tom and Mary, were trying to rendezvous on a circle drawn on the plane, we would not allow the solution where Tom proceeds clockwise and Mary counterclockwise. The symmetric solution could be written in a book on rendezvous so that

S. Alpern (✉)
ORMS Group, Warwick Business School, University of Warwick, Coventry CV4 7AL, UK
e-mail: steve.alpern@wbs.ac.uk

S. Alpern et al. (eds.), *Search Theory: A Game Theoretic Perspective*,
DOI 10.1007/978-1-4614-6825-7__14, © Springer Science+Business Media New York 2013

when two people find themselves faced with the rendezvous problem they could read what to do in this book, without having decided beforehand on which roles (e.g. clockwise or counter-clockwise) to play. The asymmetric version allows Tom and Mary to agree beforehand on which roles to take: for example one could wait while the other searches (the *Wait for Mommy* strategy).

14.2 The Problems

We now list some rendezvous problems which are unsolved. When partial solutions, or solutions to special cases have been found, we will mention this afterwards.

1. **The astronaut problem.** Two unit speed astronauts land simultaneously on a small smooth sphere. They walk at the same speed and can detect each other when they are a given distance apart. What is the least expected meeting time they can guarantee, and what strategies should they use to achieve this time?

As far as we are aware, no significant progress has been made on this problem. An easier version would spin the sphere, so that the two poles would be focal points. Even the latter problem seems to be open, though some sort of randomized oscillation between the poles would seem to be useful in the latter problem. But how long should one wait before heading for the other pole? A lower dimension analog of this problem is given next.

2. **Rendezvous on a circle.** Two players are uniformly and independently placed on a circle, without any common sense of direction (or of *up*, if the circle is drawn on a plane). They move at unit speed and must use the same mixed strategy. Determine the least expected meeting time.

It has been conjectured by the author that the solution of this problem is for both players to use the so called cohato (coin half tour) strategy: oscillate between your starting point and its antipode, each time choosing equiprobably and independently of prior choices between your clockwise and your counter-clockwise (or simply choose two directions and do one if Heads and the other if Tails on the coin). Simpler versions of this problem, where the players have some additional information or common notions, have been solved by Howard [22] and Alpern [4, 6].

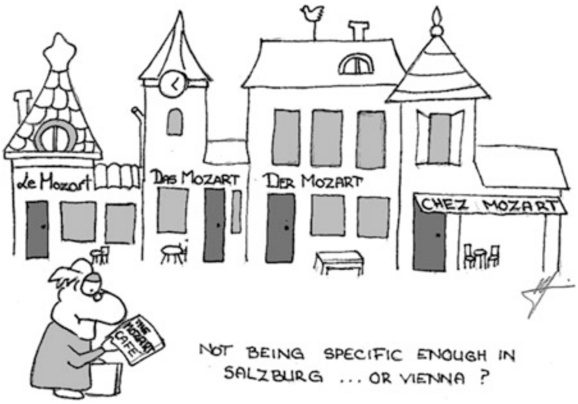
3. **Rendezvous on the infinite line (agent-symmetric form).** In this version of the rendezvous problem, the two players are randomly placed on a common line and try to find each other. Even for the apparently simpler version where the initial distance (taken for simplicity as 2) is known, the problem is open.

It has been shown by Lim, Alpern and Beck [27] and Qiaoming, Du, Vera and Zuluaga [31] that the players can optimally restrict their mixing to strategies which move at unit times along the integer lattice determined by their starting point. So anyone trying to solve this problem can restrict their search to these simple strategies.

The author has recently established by a compactness argument that the least expected time and optimal strategies exist (not just $\varepsilon-$optimal ones). The author initially [2, 3] suggested the strategy where in each time period of length 3 the players independently chose a forward direction and then move forward, back, back. This strategy is easily seen to have expected meeting time 5/2. It has been bettered by strategies using longer sequences of moves, by Anderson and Essegaier [15], Baston [17], Uthaisombut [34], Alpern [7] and Han et al. [31]. Currently the best estimates combine to give $2.091 \leq R^s \leq 2.1287$.

4. **Rendezvous on the infinite line (agent-asymmetric form).** In this version of the problem, the cumulative distribution function $F(d)$ of the initial distance d between the players is given. In general, this is an easier problem, though a solution for general F is not known.

In the case where the initial distance d is given, the solution was found by Alpern and Gal [11], and the least expected meeting time is $13d/8$. Further progress was made when Alpern and Howard [13] showed that the problem was equivalent to a single agent problem where a single searcher seeks to find an object hidden at one of two possible locations, where each location must be searched in a given order and alternation between locations is costless (the *alternating search problem*). Using this approach Alpern and Beck [10] were able to solve the problem for the case where d has a convex distribution on an interval $[0, D]$. In these cases the solution is a variation on the so called *Wait For Mommy* strategy, where one player (Baby) waits while the other carries out an optimal search for an immobile hider. In the variation, Mommy doesn't change her strategy, but Baby moves to meet Mommy. These solutions don't apply to the symmetric problem because they require coordination in the assignment of roles (Baby, Mommy) to the players.



5. **Mozart Café problem.** Two friends agree to meet for lunch on the first day of the millennium year 2,000 at the Mozart Café in Vienna. When the time comes, each arrives at Vienna Airport and asks a cab to take them to the Mozart Café. Each is troubled to hear the answer 'There are four of them; Which one do you want?' On the first day the four cafes are indistinguishable so each can do no better than picking one at random. If they don't meet on the first day (1 January) then they can choose to return on day 2 to the same cafe, or to go to a random new one. And so on. What is the best strategy, assuming both players use the same one (with independent randomization)?

This is the first discrete time rendezvous problem. It remains open if there are at least $n = 4$ cafes. If there are only $n = 2$ cafes, Anderson and Weber [16] showed that the random strategy (pick randomly every day) is optimal. They proposed a general strategy $AW(n)$ for the case of n cafes: If you haven't met on day 1, do the following in every successive interval of $n-1$ days; with probability p_n, search the other $n-1$ cafes in random order; with probability $1-p_{n-1}$, stay where you are for another $n-1$ days. Weber [35] has recently used an elegant but elaborate argument to show that $AW(3)$ is indeed optimal for the 3 cafe problem, but that $AW(4)$ is not optimal for $n = 4$.

6. **Mozart Cafe problem with river.** Suppose there are $2n$ cafes, with n of them on each side of the Danube. The problem has changed, because after not meeting on day 1 for example, a player has three choices for the next day: the same cafe, a different one on the same side of the river, a random cafe on the other side of the river.

Nothing is known about this version of the problem. The players can ignore the additional information, so they should be able to meet in the same expected time as the regular $2n$ problem. But can they do better? More generally, we could have $n = mr$ cafes partitioned into m sets of r. (Unequal partitions could allow coordination on certain sets.)

7. **Multiple agent rendezvous.** The rendezvous problems can be modified so that the goal is for n players to all meet at the same location. For example, n astronauts could land independently with the same distribution on the sphere.

For these problems it could be assumed that players who meet must stick together (sticky) or not. One could also look at problems where there are n players but only k of them have to meet (say, to carry out some task). Some references to various versions of this problem (including some in the computer science literature which we have been otherwise excluding) are Alpern [6], Lin, Morse, and Anderson [28, 29], Baston [17], Lim, Alpern and Beck [27], Alpern and Lim [14, 26], Dessmark, Fraigniaud, Kowalski and Pelc [20], Marco, Gargano, Kranakis, Krizanc and Pelc [30], Kranakis, Krizanc and Rajsbaum [25],Kowalski and Malinoski [24], Dobrev, Flocchini, Prencipe and Santoro [21].

8. **Asynchronous rendezvous.** All of the above problems assume that the two (or more) players enter the search region, and begin their search, at the same time. This allows some coordination.

This problem has received some attention in unpublished work of V. Baston and A. Beck, and in the alternative optimization criteria of the theoretical computer science approach of Marco, Gargano, Kranakis, Krizanc and Pelc [30]. See also Lin, Morse and Anderson [29].

9. **Rendezvous without proximity.** The classical form of the rendezvous search problem assumes that the problem is solved (rendezvous achieved) when the two players meet spatially. That is, when they come within a specified distance or, in one dimensional scenarios, when they have the same location. However, other end conditions are possible. More generally we could posit a subset R of $Q \times Q$, where Q is the search region, where the game ends when the locations of the two players form a pair in R. The classical version constitutes the diagonal.

The only problem of this type that has been explored in the literature is by the author [8], who considered a rendezvous problem in Manhattan, where the two players rendezvous when they arrive at a common street or avenue (and thus can see each other without buildings coming between them). It would be useful to develop a general theory, or perhaps simply explore another example.

10. **When to give up: rendezvous with failure.** As stated in the Introduction, what distinguishes rendezvous search from Schelling's coordination problems is that the search continues until coordination (meeting) is achieved. However it is common that when two people agree to meet in a large area, one or more may eventually give up, assuming the other didn't come to the area. Similarly, searches for missing people are eventually terminated.

There are two ways we could incorporate giving up the search: we could keep the classical formulations but change the cost function; or we could put in new ingredients to the problem. For the former, we could postulate a cost function $c(T) + f \cdot C$, where c is an increasing convex function of search time T and C is a cost of failure that is incurred if the search is terminated at a time T without meeting (in this case $f = 1$; otherwise $f = 0$). Or we could have a value of meeting which decays exponentially, while the cost is still the search time T. The later version involves changing the ground rules of the rendezvous problem itself. We could have a given probability that each player simply does not turn up to the game. Or this could be implicit: for example the initial distribution of the players could be uniform on the union of disjoint complete graphs of size 8 and 2. Presumably one would start off in the hope that the other is in the same component and play optimally within your component. At some point (earlier if you are in the small component), you have a sufficiently low updated probability that rendezvous is possible, and you would stop. If the players stop at different times it is not clear how to allocate costs to search times.

14.3 Further Comments

For more results in the field of the operations research aspects of rendezvous, see the monograph of the author and S. Gal [12] and the survey of the author [5]. For a broader approach to search games and search problems, see the monograph of Stone [33] and the survey of Benkoski, Monticino and Weisinger [18].

References

1. Alpern, S. Hide an seek games. Seminar, Institut fur Hohere Studien, Vienna, 26 July 1976.
2. Alpern, S. The rendezvous search problem. *LSE CDAM Research Report* 1993 **53,** London School of Economics.
3. Alpern, S. The rendezvous search problem. *SIAM J. Control & Optimization* 1995; **33**:673–683.
4. Alpern, S. Asymmetric rendezvous search on the circle. *Dynamics and Control* 2000; **10**:33–45